Short Papers

Step Response of Lossless Nonuniform Transmission Lines with Power-Law Characteristic Impedance Function

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Abstract —The step-response waveform of the lossless nonuniform transmission line possessing a characteristic impedance function $Z_c(x) = Z_0(1+\eta x)^{2n}$, $n = 0,1,2,\ldots$, is deduced. The simple and closed-form solutions should be useful for pulse transient analysis involving nonuniform transmission lines.

I. Introduction

Nonuniform transmission lines (NTL's) are used, e.g., as impedance matching devices [1]–[4], pulse transformers [9]–[12], filters [5], resonators [2], directional couplers [6], etc. Their frequency-domain behavior has been investigated extensively for a number of special NTL types. In certain cases, it is desirable to find exact and closed-form solutions for the *transient* response of NTL's. Even though a number of different methods were developed in the past (see, e.g., [10]), the approach via the Laplace transform seems to be one of the most suitable in achieving the above goal. Note that the availability of time-domain solutions by this method depends strongly on the kind of limit conditions (source and load impedances, Z_s and Z_l, respectively) imposed onto the system [10]. To the authors' best knowledge, closed-form time-domain solutions so far have not been reported at all, for any class of NTL's, except for a single special case [11]. In fact, the calculation of the transient response of NTL's tends to become a very involved computational problem. Reference [9] illustrates well the degree of complexity encountered in deriving the expressions for step response of the exponential transmission line (ETL).

The present analysis aims to deduce the expressions for the *step response* of a whole class of NTL's characterized by the characteristic impedance function $Z_c(x) = Z_0(1+\eta x)^{2n}$, $n = 0,1,2,\ldots$, hereafter referred to as the power-law transmission line (PLTL). Frequency-domain solutions for this type of transmission line have already been presented, e.g., by [4], [7]–[8], where exact network functions are also given. The parabolic transmission line (PTL) analyzed by the authors in [11] with index $n = 1$ is contained as a special case in this time-domain analysis. On the other hand, the ETL treated in numerous papers (frequency-domain treatment) is reached asymptotically by this class of lines, if $n \to \infty$.

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II. Basic Solutions

The analysis is based on the following assumptions for the characteristic impedance $Z_c(x)$ and the propagation function $\gamma(p)$:

$$\left.\begin{aligned} Z_c(x) &= Z_0(1+\eta x)^{2n}, \qquad n = 0,1,2,\ldots \\ \gamma(p) &= p/c \end{aligned}\right\} \tag{1}$$

where

- x position along the line,
- η taper parameter,
- p Laplace transformation complex variable,
- c wave propagation velocity (= constant).

Using (1), one finds as solutions of the well-known second-order differential equations [1]–[3], [9] describing the propagating voltage $V(x,\gamma)$ and current $I(x,\gamma)$, [12], [13]

$$V_\pm(x,\gamma) = \exp(\mp\gamma x)\, y^n \sum_{k=0}^{n} (\pm 1)^k \frac{(n+k)!}{k!(n-k)!}\left(\frac{\eta}{2\gamma y}\right)^k$$

$$I_\pm(x,\gamma) = \frac{\pm\exp(\mp\gamma x)}{Z_0} y^{-n} \sum_{k=0}^{n-1} (\pm 1)^k \frac{(n+k-1)!}{k!(n-k-1)!}\left(\frac{\eta}{2\gamma y}\right)^k \tag{2}$$

where $y = (1+\eta x)$. The suffix $\pm$ denotes waves traveling in the $+x$-direction and $-x$-direction, respectively. By setting $n = +1$, one may verify the solutions for the PTL given by [11] ($n = 0$ corresponds to the solutions of the uniform transmission line).[1]

III. Step-Response Waveform

We are interested in the waveform $f(l,t)$, which appears across the load Z_l (see configuration shown in Fig. 1), if the transmission line is excited by a unit step generator at its input. Furthermore, the time t shall be limited to the range $l/c < t < 3l/c$, where l/c is the time delay of the line and $3l/c$ corresponds to the time after which the first reflection returns from the sending end. One then obtains, using (2) and evaluating it for the limit conditions at $x = 0$ and $x = l$ (Fig. 1), the Laplace transform $F(l,p)$ [12]

$$F(l,p) = \frac{2\nu_2 y_1^n}{(1+\nu_1)(y_1^{2n}+\nu_2)} \cdot \frac{p^{2n-1}\exp(-pl/c)}{P_1(x=0,p)\,P_2(x=l,p)} \tag{3}$$

[1]Here, n is a positive integer number. For negative n, one obtains by symmetry considerations of the differential equations for voltage and current: $V_\pm(x,p;\, n<0) = \pm Z_0 I_\pm(x,p;\, n>0)$ and $I_\pm(x,p;\, n<0) = \pm(1/Z_0)V_\pm(x,p;\, n>0)$

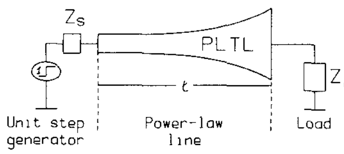

Fig. 1. Basic configuration analyzed.

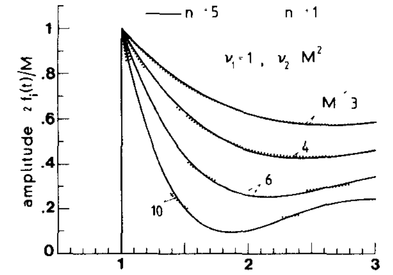

Fig. 2. Calculated step-response curves $f_t(t) = f_t(l, t)$ as function of the normalized time ct/l with the voltage transforming ratio M and the power index n as parameters. The following conditions are assumed: $\nu_1 = Z_s/Z_0 = 1$ and $\nu_2 = Z_l/Z_0 = M^2$.

where

$$\nu_1 = Z_s/Z_0 \quad \nu_2 = Z_l/Z_0 \text{ and } y_1 = (1+\eta l)$$

$$P_1(x=0,p) = \frac{1}{1+\nu_1}[V_+(0,p) + Z_s I_+(0,p)]\,p^n$$

$$P_2(x=l,p) = \frac{y_1^n}{(y_1^{2n}+\nu_2)}[V_-(l,p) - Z_l I_-(l,p)]\,p^n \exp(-pl/c) \tag{4}$$

where P_1 and P_2 are polynomials of order n in p, and V_+, V_-, I_+, I_- are the basic solutions given in (2).

If we denote the zeros of P_1 by $p_1, \ldots, p_n$ and the zeros of P_2 by $p_{n+1}, \ldots, p_{2n}$, then the step-response waveform $f(l,t)$ results in

$$f(l,t) = \frac{2\nu_2 y_1^n}{(1+\nu_1)(y_1^{2n}+\nu_2)} \sum_{k=1}^{2n} \alpha_k \exp[p_k(t - l/c)], \qquad \frac{l}{c} < t < \frac{3l}{c}. \tag{5}$$

The expression for the coefficient α_k is given by

$$\alpha_k = \frac{p_k^{2n-1}}{\prod_{\substack{m=1\\ m\neq k}}^{2n} (p_k - p_m)}. \tag{6}$$

The procedure involved in finding the step-response waveform for a given power index n can consequently be summarized into the following important steps: Calculation of a) polynomials P_1 and P_2 via (4), b) zeros of P_1 and P_2, and c) the step-response waveform $f(l,t)$ using (5) and (6).

In Figs. 2 and 3, several step-response curves are graphically illustrated for different voltage transforming ratios $M = y_1^n$ and for two different values of the generator impedance Z_s. In addition, in Fig. 3, the PLTL is compared with the exponential transmission line (ETL) analyzed by Schatz [9].

IV. Zeros of $P_1(p)$ and $P_2(p)$

One can show that all zeros of the polynomial P_1 are in the left-half p-plane ($\mathrm{Re}(p) < 0$), while all zeros of P_2 are situated in the right-half p-plane ($\mathrm{Re}(p) > 0$) [12].[2] In Table I, the zeros are calculated for the generator impedance $Z_s = 0$ and $Z_s = Z_0$. In both cases, $Z_l = Z_0 y_1^{2n}$ (matched configuration at the load).

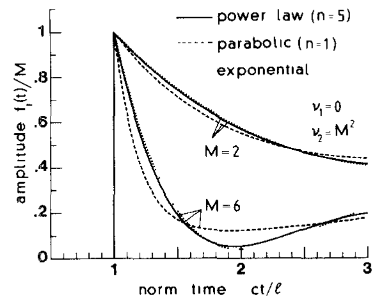

Fig. 3. Comparison of calculated step-response waveforms for the ETL, PTL, and PLTL ($n = 5$) for two values of the voltage transforming ratio M. The following parameters are assumed for the calculation: $\nu_1 = Z_s/Z_0 = 0$ and $\nu_2 = Z_l/Z_0 = M^2$. (Note: The evaluation of the step-response waveform for the ETL under arbitrary limit conditions Z_s and Z_l is rather complex. The curves for the ETL in this figure are taken from [9].)

TABLE I
ZEROS OF POLYNOMIALS P_1 AND P_2 FOR TWO DIFFERENT VALUES OF THE GENERATOR IMPEDANCE $Z_s = 0$ AND $Z_s = Z_0$.

	Zeros p_k of polynomial P_1; $k=1,\ldots,n$ Note: $p_k = p_k' \eta c$		Zeros of polynomial P_2; $k=n+1,\ldots,2n$; $p_k = p_k^* \eta c/y_1$
n	$\nu_1 = Z_s/Z_0 = 0$	$\nu_1 = Z_s/Z_0 = 1$	$\nu_2 = Z_l/Z_0 = y_1^{2n} = M^2$
1	$p_1' = -1$	$p_1' = -\frac{1}{2}$	$p_2^* = \frac{1}{2}$
2	$p_{1,2}' = -3/2 \pm j\sqrt{3}/2$	$p_{1,2}' = -1 \pm j/\sqrt{2}$	$p_{3,4}^* = 1 \pm j/\sqrt{2}$
3	$p_1' = -2.322$ $p_{2,3}' = -1.839 \pm j\,1.754$	$p_1' = -1.819$ $p_{2,3}' = -1.341 \pm j\,1.525$	$p_4^* = 1.819$ $p_{5,6}^* = 1.341 \pm j\,1.525$
4	$p_{1,2}' = -2.896 \pm j\,0.867$ $p_{3,4}' = -2.104 \pm j\,2.657$	$p_{1,2}' = -2.394 \pm j\,0.784$ $p_{3,4}' = -1.606 \pm j\,2.387$	$p_{5,6}^* = 2.394 \pm j\,0.784$ $p_{7,8}^* = 1.606 \pm j\,2.387$
5	$p_1' = -3.647$ $p_{2,3}' = -3.352 \pm j\,1.743$ $p_{4,5}' = -2.325 \pm j\,3.571$	$p_1' = -3.143$ $p_{2,3}' = -2.850 \pm j\,1.605$ $p_{4,5}' = -1.828 \pm j\,3.272$	$p_6^* = 3.143$ $p_{7,8}^* = 2.850 \pm j\,1.605$ $p_{9,10}^* = 1.828 \pm j\,3.272$

In both cases, a load impedance $Z_l = Z_0 y_1^{2n} = Z_0 M^2$ is assumed (the load impedance is matched to the impedance level of the line at $x = l$). The zeros for any other values of Z_s and Z_l can be calculated using (4).

V. Conclusions

The extension of the step-response analysis for the parabolic transmission line ($n = 1$) to the more general case of the characteristic impedance function $Z_c(x) = Z_0(1+\eta l)^{2n}$, $n = 0, 1, 2, \ldots$, expands the class of nonuniform transmission lines having a simple step-response waveform. The model should consequently be useful for transient analysis involving nonuniform transmission lines. Because of the relationship

$$Z_c(x) = Z_0 \exp(\eta x) = \lim_{n\to\infty} Z_0 \left(1 + \frac{\eta x}{2n}\right)^{2n}$$

such a power-law transmission line may be used to approximate the step response of the exponential line. Fig. 3 shows this approximation behavior. As may be seen from [9], such an approximation function facilitates considerably the quantitative evaluation of the step response waveform (at least for reasonably low values of the power index n).

[2] The reader should not get confused by the fact that the zeros of the polynomial P_2 are in the right-half p-plane ($\mathrm{Re}(p) > 0$). The stability of the system is still assured, as the time variable t was limited to $t < 3l/c$ (see (5)).

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Development and Testing of a 2450-MHz Lens Applicator for Localized Microwave Hyperthermia

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***Abstract*—A new type of applicator with a convergent lens for localized microwave hyperthermia is developed. A lens applicator of direct contact type was designed to conduct actual and progressive experiments with phantoms of simulated fat and muscle tissues heated at 2450 MHz. The experimental results showed that the heating power penetration depth increased 40 percent with this applicator as compared to a simple rectangular waveguide applicator with the same size aperture that had generally been used for microwave hyperthermia. Our applicator had a concave-shaped aperture and was designed to contact well with the heating medium whose shape was cylindrical like a human body.**

I. Introduction

The development of noninvasive localized heating techniques for the human body is indispensable for hyperthermia. Dielectric heating by electromagnetic (EM) waves is one of the best means for providing these heating techniques. EM techniques and applicators for medical diagnosis and therapy have recently been observed [1]–[3]. To perform effective hyperthermia, the design of an applicator to transfer EM energy to the treatment area is an important problem. When microwave EM fields are used, the depth of penetration is generally shallow and it is difficult to heat deep-lying tissues and relatively large tissue volumes. To overcome this difficulty, many different applicators have been developed [4]–[7]. The desired characteristics of a direct contact applicator for microwave hyperthermia are to deposit EM energy effectively in the defined tissue volume, to have a good impedance matching, and to be easy to handle. In view of these characteristics, hyperthermia applicators still have much room for improvement [8].

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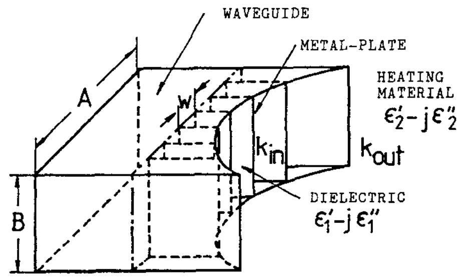

Fig. 1. Schematic of lens applicator of direct contact type.

The excellent features of this new type of applicator with a convergent lens, designed by geometrical optics and a concave-shaped aperture that provides good contact with the cylindrical-shaped human body, have been previously presented [9]–[11].

Our plan of presentation is as follows. In the second section, we propose a new lens applicator for hyperthermia in order to deposit EM field energy inside the deep medium, and carry out heating experiments using phantom modeling material for human tissues. We calculate theoretically the electric field distribution from the applicator. We compare our results with the electric field distribution obtained from traditional waveguide. Finally with our apparatus, we present and discuss the results.

II. Development of Lens Applicator

A. Design Principle of the Lens Applicator of Direct Contact Type

Assume that a parallel metal-plate medium with plate distance w and filled with a dielectric material with complex dielectric constant $\epsilon_1' - j\epsilon_1''$ ($=\epsilon_1^*$) is inserted into a waveguide so that the metal plates are in parallel to the E-plane (see Fig. 1). Letting λ' be the wavelength of the EM wave in the dielectric material, the EM wave in the metal-plate medium has the propagation mode TE_{10} for the range of a constant separation w between each pair of the metal plate satisfying $\lambda'/2 < w < \lambda'$. The propagation constant k_{in} then is given by

$$k_{\text{in}} = \sqrt{\omega^2 \mu (\epsilon_1' - j\epsilon_1'') - \left(\frac{\pi}{w}\right)^2}. \tag{1}$$

When such a waveguide applicator, with the metal-plate medium filled with dielectric material, is made to contact to another dielectric material at the aperture, the EM wave refracts at the boundary of dielectric materials, as is seen from the theory of geometrical optics. Letting the complex dielectric constant of the dielectric material in contact with the applicator be $\epsilon_2' - j\epsilon_2''$ ($=\epsilon_2^*$) (see Fig. 1), the propagation constant of the medium k_{out} is given as in

$$k_{\text{out}} = \beta - j\alpha \tag{2}$$